Logarithmic differentiation

To differentiate $y = f(x)$, it is often easier to use logarithmic differentiation :

1. Take the natural logarithm of both sides to get $\ln y = \ln(f(x))$.
2. Differentiate with respect to x to get $\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\ln(f(x))$.
3. We get $\frac{dy}{dx} = y\frac{d}{dx}\ln(f(x)) = f(x)\frac{d}{dx}\ln(f(x))$.

Logarithmic differentiation; Example

Find the derivative of $y = \sqrt[4]{\frac{x^2+1}{x^2-1}}$.

- We take the natural logarithm of both sides to get
$$\ln y = \ln \sqrt[4]{\frac{x^2+1}{x^2-1}}$$
- Using the rules of logarithms to expand the R.H.S. we get $\ln y = \frac{1}{4}\ln\frac{x^2+1}{x^2-1} = \frac{1}{4}\Big[\ln(x^2+1) - \ln(x^2-1)\Big] = \frac{1}{4}\ln(x^2+1) - \frac{1}{4}\ln(x^2-1)$
- Differentiating both sides with respect to x, we get $\frac{1}{y}\frac{dy}{dx} = \frac{1}{4}\cdot\frac{2x}{(x^2+1)} - \frac{1}{4}\cdot\frac{2x}{(x^2-1)} = \frac{x}{2(x^2+1)} - \frac{x}{2(x^2-1)}$
- Multiplying both sides by y, we get $\frac{dy}{dx} = y\Big[\frac{x}{2(x^2+1)} - \frac{x}{2(x^2-1)}\Big]$
- Converting y to a function of x, we get
$$\frac{dy}{dx} = \sqrt[4]{\frac{x^2+1}{x^2-1}}\left[\frac{x}{2(x^2+1)} - \frac{x}{2(x^2-1)}\right]$$

Summary of Formulas for Logarithms

$$\boxed{\ln(x)}$$

$$\ln 1 = 0, \qquad \ln e = 1.$$

$$\ln(ab) = \ln a + \ln b, \quad \ln(\frac{a}{b}) = \ln a - \ln b,$$

$$\ln a^x = x \ln a$$

$$\lim_{x \longrightarrow \infty} \ln x = \infty, \quad \lim_{x \longrightarrow 0} \ln x = -\infty$$

$$\frac{d}{dx} \ln |x| = \frac{1}{x}, \quad \frac{d}{dx} \ln |g(x)| = \frac{g'(x)}{g(x)}$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int \frac{g'(x)}{g(x)} dx = \ln |g(x)| + C.$$